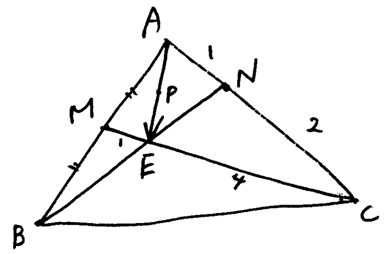
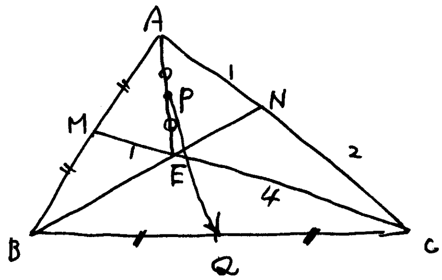


メネラウスの定理より. $\frac{AB}{BM}\cdot\frac{ME}{EC}\cdot\frac{CN}{NA}=\frac{2}{1}\cdot\frac{ME}{EC}\cdot\frac{2}{1}=1$

$\therefore\ \frac{ME}{EC}=\frac{1}{4}$

$\therefore\ \overrightarrow{AE}=\frac{4\overrightarrow{AM}+\overrightarrow{AC}}{5}=\frac{4\overrightarrow{AM}+(3\overrightarrow{AN})}{5}=\frac{4}{5}\overrightarrow{AM}+\frac{3}{5}\overrightarrow{AN}$ //



$\overrightarrow{PQ}=\overrightarrow{PA}+\overrightarrow{AQ}$

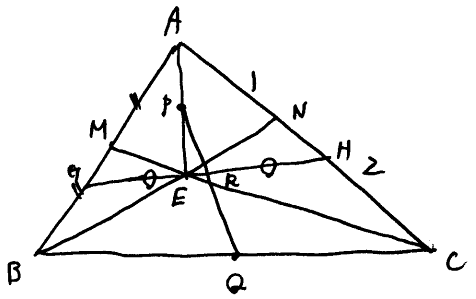
$=-\overrightarrow{AP}+\overrightarrow{AQ}$

$=-\frac{1}{2}\overrightarrow{AE}+\frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$

$=-\frac{1}{2}\left\{\frac{4}{5}\overrightarrow{AM}+\frac{3}{5}\overrightarrow{AN}\right\}+\frac{1}{2}\left\{2\overrightarrow{AM}+3\overrightarrow{AN}\right\}$

$=\left\{1-\frac{4}{10}\right\}\overrightarrow{AM}+\left\{\frac{3}{2}-\frac{3}{10}\right\}\overrightarrow{AN}$

$=\frac{6}{10}\overrightarrow{AM}+\frac{12}{10}\overrightarrow{AN}=\frac{3}{5}\overrightarrow{AM}+\frac{6}{5}\overrightarrow{AN}$ //



$\overrightarrow{AG}=x\overrightarrow{AM}$, $\overrightarrow{AH}=y\overrightarrow{AN}$

$\overrightarrow{AR}=\frac{x\overrightarrow{AM}+y\overrightarrow{AN}}{2}=\frac{1}{2}x\overrightarrow{AM}+\frac{1}{2}y\overrightarrow{AN}$

$\overrightarrow{AR}=\overrightarrow{AP}+k\overrightarrow{PQ}$ として.

右辺 $=\frac{1}{2}\{\overrightarrow{AE}\}+k\left\{\frac{3}{5}\overrightarrow{AM}+\frac{6}{5}\overrightarrow{AN}\right\}$

$=\frac{1}{2}\left\{\frac{4}{5}\overrightarrow{AM}+\frac{3}{5}\overrightarrow{AN}\right\}+k\left\{\frac{3}{5}\overrightarrow{AM}+\frac{6}{5}\overrightarrow{AN}\right\}$

$=\left(\frac{2}{5}+\frac{3}{5}k\right)\overrightarrow{AM}+\left(\frac{3}{10}+\frac{6}{5}k\right)\overrightarrow{AN}$

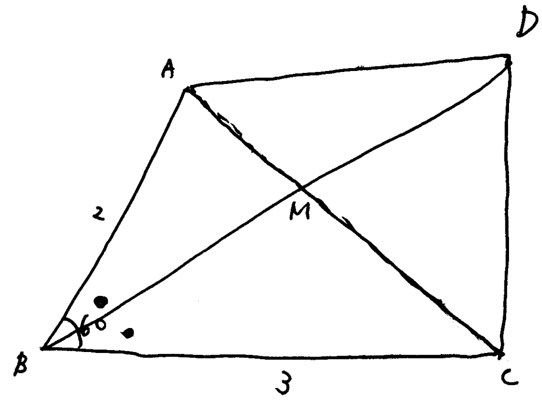
左辺＝右辺　$\frac{2+3k}{5}=\frac{1}{2}x$　　$5x=4+6k$　$\therefore\ 6k=5x-4$

$12k=10x-8$

$\frac{3+12k}{10}=\frac{1}{2}y\ \rightarrow\ \frac{3+(10x-8)}{10}=\frac{1}{2}y\ \Leftrightarrow\ \frac{10x-5}{10}=\frac{1}{2}y$

$y=\frac{10x-5}{5}$

$=2x-1$ //



(1) $\overrightarrow{AD} = -\overrightarrow{BA} + \overrightarrow{BD}$, $\overrightarrow{CD} = -\overrightarrow{BC} + \overrightarrow{BD}$

$|\overrightarrow{AD}| = |\overrightarrow{CD}|$ より. $|\overrightarrow{AD}|^2 = |\overrightarrow{CD}|^2$

$|\overrightarrow{BA}|^2 - 2\overrightarrow{BA}\cdot\overrightarrow{BD} + |\overrightarrow{BD}|^2 = |\overrightarrow{BC}|^2 - 2\overrightarrow{BC}\cdot\overrightarrow{BD} + |\overrightarrow{BD}|^2$

$4 - 2\overrightarrow{BA}\cdot\overrightarrow{BD} = 9 - 2\overrightarrow{BC}\cdot\overrightarrow{BD}$

(☺ AB : BC = 2 : 3 であるので, $|\overrightarrow{BA}| = 2$, $|\overrightarrow{BC}| = 3$ としてよい。)

$\overrightarrow{BC}\cdot\overrightarrow{BD} - \overrightarrow{BA}\cdot\overrightarrow{BD} = \frac{5}{2}$

ここで, $\overrightarrow{BD} = k\overrightarrow{BM} = \frac{3}{5}k\overrightarrow{BA} + \frac{2}{5}k\overrightarrow{BC} = 3x\overrightarrow{BA} + 2x\overrightarrow{BC}$ とおける.

$\overrightarrow{BC}\cdot(3x\overrightarrow{BA} + 2x\overrightarrow{BC}) - \overrightarrow{BA}\cdot(3x\overrightarrow{BA} + 2x\overrightarrow{BC}) = \frac{5}{2}$

$\overrightarrow{BA}\cdot\overrightarrow{BC} = \overrightarrow{BC}\cdot\overrightarrow{BA} = |\overrightarrow{BA}|\cdot|\overrightarrow{BC}|\cos 60^\circ = 3$ を利用して.

$9x + 18x - 12x - 6x = \frac{5}{2}$

$9x = \frac{5}{2}$ $\quad x = \frac{5}{18}$

$\overrightarrow{BD} = 3x\overrightarrow{BA} + 2x\overrightarrow{BC} = \frac{5}{6}\overrightarrow{BA} + \frac{5}{9}\overrightarrow{BC}$ //

(2) (1)と同様に. $|\overrightarrow{AD}| = |\overrightarrow{CD}| \Leftrightarrow |\overrightarrow{AD}|^2 = |\overrightarrow{CD}|^2$

$\Leftrightarrow \overrightarrow{BC}\cdot\overrightarrow{BD} - \overrightarrow{BA}\cdot\overrightarrow{BD} = \frac{5}{2}$

$\overrightarrow{BD} = \frac{3}{2}\overrightarrow{BE} = \frac{3}{2}\{k\overrightarrow{BC} + (1-k)\overrightarrow{BA}\} = \frac{3}{2}(1-k)\overrightarrow{BA} + \frac{3}{2}k\overrightarrow{BC}$

$\frac{3}{2}k = x$ とおくと. $\frac{3}{2}(1-k) = \frac{3}{2} - x$ $\quad\therefore \overrightarrow{BD} = \left(\frac{3}{2} - x\right)\overrightarrow{BA} + x\overrightarrow{BC}$

(1)と同様に x を求めると. $x = \frac{4}{7}$

$\frac{3}{2} - x = \frac{13}{14}$

$\therefore \overrightarrow{BD} = \frac{13}{14}\overrightarrow{BA} + \frac{4}{7}\overrightarrow{BC}$ //

[1] (1) $S_n = a \cdot \dfrac{1-r^n}{1-r}$

$S_4 = a \dfrac{1-r^4}{1-r}$ $\quad S_8 = a \dfrac{1-r^8}{1-r}$

$\dfrac{S_8}{S_4} = \dfrac{1-r^8}{1-r^4} = \dfrac{65}{45} = \dfrac{13}{9}$

$r^4 = X$ として. $\dfrac{1-X^2}{1-X} = 1+X = \dfrac{13}{9}$ $\quad X = \dfrac{4}{9}$ $\quad r^4 = \dfrac{4}{9}$ $\quad r^2 = \dfrac{2}{3} > 0$

$\therefore r = \pm\sqrt{\dfrac{2}{3}} = \pm\dfrac{\sqrt{6}}{3}$

(2) $\dfrac{S_{15}}{S_{10}} = \dfrac{1-r^{15}}{1-r^{10}} = \dfrac{37}{21}$ $\quad r^5 = X$ として,

$r^{10} = X^2,\ r^{15} = X^3$. $\quad \dfrac{1-X^3}{1-X^2} = \dfrac{(1-X)(1+X+X^2)}{(1+X)(1-X)} = \dfrac{1+X+X^2}{1+X} = \dfrac{37}{21}$

$\therefore 21(1+X+X^2) = 37(1+X) \iff 21X^2 - 16X - 16 = 0$

$\iff (3X-4)(7X+4) = 0 \iff X = \dfrac{4}{3},\ -\dfrac{4}{7}$

$\dfrac{S_{10}}{S_5} = \dfrac{1-X^2}{1-X} = 1+X = \dfrac{21}{S_5}$ $\quad \therefore S_5 = \dfrac{21}{1+X}$

$X = \dfrac{4}{3}$ の時 $S_5 = \dfrac{21}{1+\frac{4}{3}} = \dfrac{21}{\frac{7}{3}} = \dfrac{21}{7} \times 3 = 9$ //

$X = -\dfrac{4}{7}$ の時, $S_5 = \dfrac{21}{1-\frac{4}{7}} = \dfrac{21}{\frac{3}{7}} = \dfrac{21}{3} \times 7 = 49$ //